This paper presents a physics based analytical model to predict the thermal behavior of pin fin heat sinks in transverse forced flow. The key feature of the model is the recognition that unlike plate fins, streamwise conduction does not occur in pin fin heat sinks. Thus, the heat transfer from each fin depends on its local air temperature or adiabatic temperature and the local adiabatic heat transfer coefficient. Both experimental data and simplified CFD simulations are used to develop the two building blocks of the model, the thermal wake function and the adiabatic heat transfer coefficient. These building blocks are then used to include the effect of the thermal wake from upstream fins on the adiabatic temperature of downstream fins in determining the fin-by-fin heat transfer within the pin fin array. This approach captures the essential physics of the flow and heat transport within the fin array and yields an accurate model for predicting the thermal resistance of pin fin heat sinks. Model predictions are compared with existing experimental data and CFD simulations. The model is expected to provide a sound basis for a consistent performance comparison with plate fin heat sinks.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area</td>
</tr>
<tr>
<td>(A_{\text{min}})</td>
<td>minimum free flow area between pins (or pin fins)</td>
</tr>
<tr>
<td>(c_{p})</td>
<td>specific heat</td>
</tr>
<tr>
<td>D</td>
<td>diameter of pins</td>
</tr>
<tr>
<td>f</td>
<td>per-pin friction factor (\left(\frac{\Delta p/N}{\frac{1}{2}bU_o^2}\right))</td>
</tr>
<tr>
<td>h</td>
<td>heat transfer coefficient</td>
</tr>
<tr>
<td>i,j,k</td>
<td>index variables</td>
</tr>
<tr>
<td>L</td>
<td>length of pin</td>
</tr>
<tr>
<td>M</td>
<td>number of rows in the pin fin array</td>
</tr>
<tr>
<td>N</td>
<td>number of pins in a row</td>
</tr>
<tr>
<td>Nu</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>O</td>
<td>number of segments on each fin</td>
</tr>
<tr>
<td>p</td>
<td>pressure</td>
</tr>
<tr>
<td>P</td>
<td>perimeter</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>q</td>
<td>heat transfer rate</td>
</tr>
<tr>
<td>Q</td>
<td>heat input at boundary</td>
</tr>
</tbody>
</table>

\(\Delta p\) | difference in pressure |

Greek Symbols

\(\lambda\) | thermal conductivity |
\(\mu\) | viscosity |
\(\rho\) | density |
\(\eta\) | fin efficiency |
\(\theta\) | wake function or normalized adiabatic temperature rise |
\(\chi\) | friction factor ratio for non-square arrays \((S_L/D)\) |

Subscripts

a | air or approach |
ad | adiabatic |
f | fin |
in | air inlet conditions |
m | metal thermophysical property |

INTRODUCTION

Air cooled heat sinks are the workhorse device for all sorts of components in the electronics industry. The widespread availability of user-friendly computational fluid dynamics (CFD) software now enables heat sink designers and to obtain a reasonably accurate prediction of heat sink performance. Although desktop computers have become powerful enough to run CFD models, a full parametric design study using CFD alone remains impractical because solutions times are still measured in hours and post processing analysis of large CFD data sets remains time consuming. Simple accurate numerical models for heat sinks remain highly desirable to expedite design trade-offs and optimization analyses, e.g. [1-3].
Given a set of design constraints and an appropriate heat sink model, a multi-parameter optimization can be carried out using the procedure illustrated by Culham and Muzychka [3]. The derivatives of the minimization function with respect to each independent parameter in [3] can be determined from a function call to a heat sink model with any desired level of complexity. The accuracy of the design optimization thus achieved will depend on how faithfully the model embodies the physics governing heat transport within the heatsink structure.

Numerous investigations in the literature have been directed at parallel plate heat sinks in transverse flow e.g. Iwazaki et. al. [4], Teerstra et. al. [5], Lee [6] using fin efficiency calculations based on a duct length averaged Nusselt number. We call this a one-dimensional (1-D) approach since the fin temperature contours are implicitly approximated as horizontal isotherms. The flowstream based model described by Holahan et. al. [7] uses the two-dimensional (2-D) temperature field in the fin to determine heat transfer and enables alternate flow paths such as top-inlet-side exit to be modeled as in [8].

For pin fin heat sinks, the information available is relatively sparse. A large experimental data set for cross-cut pin fin heat sinks by Shaukatullah [9] used a test configuration with considerable flow bypass but the pressure drop across the heat sink was not reported. Jonsson and Palm [10] tested heat sinks with plate fins, square pin fins, circular pin fins, and elliptical pin fins and reported both thermal resistance and pressure drop. However, their setup had large bypass and the pressure taps were too far upstream and downstream of the heat sink to correctly measure its driving pressure drop. Recent research undertaken by Ortega’s group whose initial results are reported by Dogruoz et. al. [11] shows promise for generating detailed pressure drop data that can be used to develop accurate design models.

Because of the limitations of the available pin fin heat sink data, most designers turn to the well documented heat transfer and friction factor correlations developed by Zukauskas [12] for tube banks in cross-flow. Jonsson and Moshfegh [13] provide new experimental data that was used in the present work to extend the geometry range to sparser pin fin arrays. A common modeling approach is to use the heat transfer coefficient from the Zukauskas correlations in an equation of the following form:

$$\theta_{sa} = \frac{Q}{h \cdot \Delta T} + \frac{\phi Q}{m \cdot c_p},$$

where $\phi \leq 1$ is a multiplier whose value depends on the reference temperature difference used as the basis for the heat transfer coefficient $h$.

For example, $\phi = 0.5$ for a uniform temperature boundary condition at the base where the driving temperature difference would be the average bulk temperature of the fluid in the array. We again designate this as a “1-D” approach since the variation of the fluid bulk temperature, fin temperature and fin efficiency along the flow direction is not accounted for.

A considerable improvement in the accuracy of heat transfer prediction from the heat sink could be made by accounting for the “2-D” nature of the fluid and pin fin temperature distribution in the pin fin array (this still ignores “3-D” flow patterns at the base and tips of the fins). A further improvement could be made by recognizing and accounting for the real driving potential for convective heat transfer between the fin and the fluid: namely the fin surface temperature to fin adiabatic temperature difference. Knowledge of the thermal wake in the pin fin array is required to predict fin adiabatic temperatures. The objective of the present work is to describe such a model by extending the approach developed in Holahan et. al. [7]. While the modeling approach presented here is applicable to heat sinks with a variety of pin fin shapes including circular, rectangular, elliptical, interrupted plate etc., appropriate heat transfer and pressure drop models are required in each case. In this work, we use empirical correlations and CFD simulations with a commercial CFD code, Icepak [14], to develop a model for heat sinks with in-line arrays of circular pin fins.

**DEVELOPMENT OF THE ANALYTICAL MODEL**

The heat transport within the type of heatsink structures considered in this work differs from that of plate fin heatsinks because of the absence of streamwise heat conduction within the fins. Heatsinks in this category include a variety of pin fin and interrupted plate fin designs like those illustrated in Figure 1. To correctly represent the fluid and heat transport in these heatsinks the model needs to include the following:

- path taken by the fluid streams through the heatsink
- pressure gradient due to frictional and form drag
- heat transfer coefficient on the wetted surfaces of the heat sink
- 1-D heat conduction within each fin from the base to the tip
- adiabatic temperature rise of a fin due to heat added to each fluid stream by upstream fins
- heat conduction within the base of the heatsink, and
- thermal boundary condition applied to the base of the heatsink

**Figure 1. Pin fin and Interrupted plate fin heat sinks**

In the present work we consider transverse laminar flow through a fully ducted heat sink with Reynolds number ranging from 10 to 3000. We restrict our attention to in-line arrays of circular pin fins since a suitable body of empirical data and correlations are available in the literature. Figure 2 depicts the pin fin array geometry considered.

**Figure 2. Circular pin fin array studied**
The development of the heat sink model is presented in two steps. In the first step we use CFD models to illustrate the thermal wake within an array of fins and to show the grid density that is required so that CFD predictions reasonably match the existing empirical data for pressure gradient and heat transfer. We use the CFD results and empirical data to develop correlation equations for the heat transfer coefficient, the pressure drop, and the wake function that are needed as building blocks for the heat sink model. In the second step we describe the framework of the pin fin heat sink model that captures the essential physics of the flow and heat transport within the fin array.

Building Block Correlations

The first CFD model was for a 3 x 8 array of circular pins situated in a duct. The longitudinal and transverse pitch to diameter ratio of the array was 2.0 and the distance between the side walls of the duct and the centers of the nearest row of pins was half the transverse pitch. A uniform air velocity and temperature were set at the duct entrance while an outflow boundary condition was specified at the duct exit. Zero velocity gradient and adiabatic thermal boundary conditions were applied at the duct walls to simulate a large array in the transverse direction. All the pins were modeled with a thermal conductivity of 180 W/m-K and zero heat dissipation rate except for the second pin in the center row which had a finite heat dissipation rate. Air was modeled with constant thermophysical properties as documented in Table 1.

<table>
<thead>
<tr>
<th>TABLE 1. Air Properties Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, ( \rho )</td>
</tr>
<tr>
<td>Thermal Conductivity, ( \lambda_a )</td>
</tr>
<tr>
<td>Viscosity, ( \mu )</td>
</tr>
<tr>
<td>Specific Heat, ( c_p )</td>
</tr>
</tbody>
</table>

Figure 3 presents the adiabatic temperature rise of each of the unheated pins in the array normalized by the bulk air temperature rise for the row with the heated pin using the “per row heat capacity” as follows:

\[
\theta_{pin} = \frac{(T_{pin} - T_{in})}{\Delta T_{bulk}} \quad (1)
\]

\[
\Delta T_{bulk} = \left( \frac{q_{pin}}{\rho U_a c_p S_d L} \right) \quad (2)
\]

Figure 3 shows that only those pins located in the same row as the heated pin experience a measurable temperature rise, i.e., the thermal wake is confined to the transverse flow stream containing the heated pin. The magnitude of the adiabatic temperature rise of the pin immediately downstream of the heated pin is considerably greater than the bulk temperature rise of the air in the heated row. Further downstream of the heated pin, the adiabatic temperature rise decays towards the bulk temperature rise. The decay is essentially complete (within 5 percent) by about five pins downstream of the heated pin. This variation of the adiabatic temperature rise is termed the “thermal wake function”. An interesting finding is the small temperature rise experienced by the unheated pin upstream of the heated pin. This is caused by flow recirculation in the wake region of each pin.

A grid sensitivity study was conducted on the geometry with \( X_L=2 \) and \( X_T=2 \) at a Reynolds number of about 750. The initial grid used was representative of the grid density that is commonly used by CFD users in industry when they model complete heat sinks. For each grid density the per pin friction factor and Nusselt number for the heated pin were determined. Grid density is calculated as the product of the ratios of pin diameter to maximum grid size specified in the x and y directions. The Nusselt number is calculated using the following definition:

\[
Nu = \left( \frac{q_{pin}}{\pi DL(T_{pin} - T_{in})} \right) \frac{D}{\lambda_a} \quad (3)
\]

The grid sensitivity results are reported in Figure 4. These results imply that a very fine grid is necessary to accurately model the behavior of pin fin heat sinks. Such a fine grid is rarely used in most applications of CFD within the electronics cooling industry. Therefore, heat sink models of the kind developed here which embody the results from fine
grid CFD simulations will provide a more accurate prediction of fully ducted heat sink behavior than most practical CFD models of the complete heat sink.

Figure 4. Grid-sensitivity of friction factor and Nusselt number

A CFD model of the single row of pins with the grid density of 225 from Figure 4 was used to develop the building block correlations for the analytical model. While not entirely grid independent, the forthcoming discussion will show that the CFD models strike a reasonable balance between providing good agreement with empirical data and economy of computational effort.

Results obtained from the CFD simulations include the per-row friction factor, Nusselt number, and the thermal wake function. Figures 5a and 5b respectively show the per pin friction factor results for square and non-square arrays using the same format as Zukauskas[12]. For square arrays the per pin friction factor for each value of $X_L$ is computed as defined in the nomenclature. The friction factor ratio $\chi$ for non-square arrays is computed by dividing the friction factor result for the $X_T < X_L$ simulation by the result for the square array with the same $X_T$. As shown in Figure 5(a), the CFD results for square arrays are in good agreement with the empirical data compiled by Zukauskas over the range $X_L = X_T = 1.5 \ldots 2.5$. Results shown in Fig. 5(b) for the non-square arrays agree quite well with the empirical data for $X_T < X_L$ but are low for $X_T > X_L$. For calculating the friction factor we use equation (4) and the coefficients given in Table 3. This correlation is based on empirical data from Zukauskas [12] except for the last row in the table which is based on pin fin heatsink data from Jonsson and Moshfegh [13].

$$f = X^{-0.754} \cdot ((C1 Re^{-0.8})^m + C2^m)^{1/m} \quad 10 < Re < 3000$$

where $X = \frac{(X_T - 1)}{(X_L - 1)}$

<table>
<thead>
<tr>
<th>$X_L$</th>
<th>$C1$</th>
<th>$C2$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>48</td>
<td>0.30</td>
<td>2.5</td>
</tr>
<tr>
<td>2.0</td>
<td>23</td>
<td>0.25</td>
<td>2.0</td>
</tr>
<tr>
<td>2.5</td>
<td>12</td>
<td>0.18</td>
<td>1.7</td>
</tr>
<tr>
<td>4.3</td>
<td>9</td>
<td>0.10</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 3. Coefficients for the friction factor correlation

Figure 5. Friction factor results from single-row CFD model

Figure 6 compares the Nusselt numbers at the fourth pin in the flow direction with empirical correlations over the Reynolds number range from 10 to 3000 and the combinations of $X_L$ and $X_T$ listed in Table 2. The predictions agree very well with the empirical correlations of Zukauskas over the whole range studied except for the intermediate range $100 < Re < 1000$ where the correlation equation appears to be low probably due to a typographical error in [12]. The CFD results are well represented by slightly modifying and blending the correlations provided by Zukauskas for Reynolds number ranges above and below this intermediate range using the following equation also shown in Figure 6.

$$Nu = ((0.85Re^{0.4}Pr^{0.36})^{10} + (0.26Re^{0.63}Pr^{0.36})^{10})^{1/10}$$ (4)
This correlation is used for all the pins in the array based on a pin by pin Nusselt number study at \( Re \sim 750 \) which showed that the fourth pin data was representative of all the pins in the array within \( \pm 2 \) percent except for the first pin which was only 5 percent higher. The reasonable agreement seen between CFD and empirical results for the friction factor and Nusselt number provides support for the thermal wake function determined from the same CFD simulations.

The wake function is modeled in two parts, the first accounts for the adiabatic temperature rise of the first pin downstream of the heated pin and the second part accounts for the decay of the wake function downstream from the first pin. In order to arrive at a suitable correlation we looked for asymptotes for the wake function at low and high Reynolds numbers and then blended the two together using the method of Churchill and Usagi [15]. Following Zukauskas, we first addressed square arrays where \( X_T = X_L \) and then looked at deviations for configurations where \( X_T \neq X_L \). The resulting correlations are inspired by the underlying physics of the data rather than a multi-parameter fit to a large set of data. The wake function for the first pin is well represented by the following equation which is compared in Figure 8 with CFD results for three configurations which represent the upper and lower bound values of \( \theta_1 \) in our CFD study.

\[
\theta_1 = 1 + \varphi \left\{ 1 - \left[ 1 + (0.015Re\varphi^{1/0.7})^2 \right]^{-0.7/2} \right\} \tag{5}
\]

where

\[
\varphi = (X_L - 1)(1.2X^2 - 0.64X + 0.48) \tag{6}
\]

As shown in Figure 7, the correlation equation provides a good fit for the data over the whole range studied. The figure shows that the value of \( \theta_1 \) starts at 1.0 (i.e. adiabatic temperature rise is equal to the bulk temperature rise) at low \( Re \) where heat diffusion is dominant, increases sharply up to an \( Re \sim 200 \) as convection becomes more and more dominant and then more slowly at \( Re > 500 \) towards a high \( Re \) asymptote of \( 1 + \varphi \). A comparison between the data for the three different geometries shows that configurations with a larger transverse pitch have higher values of \( \theta_1 \). This means that with increasing transverse pitch the bulk temperature decreases much faster (as \( 1/ST \)) than the adiabatic temperature rise in the wake of the heated pin. This is just what would be expected when fluid convection dominates. Because our CFD results are not entirely grid independent the reader should note that the true magnitude of \( \theta_1 \) is somewhat greater than the values reported here.

The decay of the thermal wake at the downstream pins is well represented by the following correlation function for square and non-square arrays.

\[
\frac{(\theta_i - 1)}{(\theta_1 - 1)} = e^{-n(i-1) + (1 - X)} \tag{7}
\]

The value of the exponent \( n \) may be determined for a given longitudinal pitch by interpolation from the values tabulated in Table 4.

**TABLE 4. Coefficients for Wake Decay**

<table>
<thead>
<tr>
<th>( \bar{X}L )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5</td>
<td>1.70</td>
</tr>
<tr>
<td>2.0</td>
<td>1.05</td>
</tr>
<tr>
<td>2.5</td>
<td>0.90</td>
</tr>
<tr>
<td>4.3</td>
<td>0.85</td>
</tr>
</tbody>
</table>

**Heat Sink Model Framework**

We consider a heat sink composed of an array of circular pin fins as shown in Figure 2. The length \( L \) of each pin is constant in the \( z \) direction. The airflow is assumed to approach the pin fin array in the \( x \) direction with a uniform velocity and temperature and to remain two-dimensional within the array (no velocity in the \( z \)-direction). Thermal wake effects are limited to fins situated along the same flow stream, i.e. within the same \( y \) and \( z \) segments.
where these terms become:

From each segment of the fin. For segment $k$ of the fin:

$$T_0 - T_{in} = (T_0 - T_f) + (T_f - T_{ad}) + (T_{ad} - T_{in})$$ (8)

These separate pieces can be written in terms of the heat transferred from each segment of the fin. For segment $k$ on the $i^{th}$ fin in the flow direction, these terms become:

$$T_0 - T_f = \sum_{1 \leq l \leq k} \left( \frac{z_l}{\lambda_A} \right) q_{i,l}$$  \hspace{0.5cm} (9)

$$T_f - T_{ad} = \frac{q_{i,k}}{hPdz}$$ \hspace{0.5cm} (10)

$$T_{ad} - T_{in} = \frac{1}{\rho U_{in} c p S_f dz} \sum_{1 \leq l < i} q_{l,k} \theta_{i-l}$$ \hspace{0.5cm} (11)

where

$q_{i,k}$ is the heat transfer from segment $k$ on pin $i$

$\theta_{i-l}$ is the wake function on pin $i$ due to heat transferred from pin $l$

$z_k$ is the distance from the base to the center of segment $k$ of the fin

$dz_k$ is the length of segment $k$ of the fin

$A_j$ is the cross-sectional area of the fin ($A_j = \pi D_j^2 / 4$)

Note that value of $T_{ad}$ can be set equal to the bulk temperature by specifying $\theta_{i-l} = 1$. The adiabatic temperature rise expressed in equation (12) contains information on the path of the fluid flow through the pin fin array and the order in which the various pins are arranged along that path. The path is constrained only in the sense that it represent a flow streamtube which must be known apriori.

Equations (10) through (12) can be substituted in equation (9) to yield the heat balance equation for each segment of each fin

$$\sum_{1 \leq l \leq k} \left( \frac{z_l}{\lambda_m f} \right) q_{i,l} + \frac{q_{i,k}}{hPdz} + \frac{1}{\rho U_{in} c p S_f dz} \sum_{1 \leq l < i} q_{l,k} \theta_{i-l} - T_0 = T_{ad}$$ (12)

One more equation is needed to specify the heat input or temperature boundary condition at the base of each fin as follows:

$$S_i^q \sum_{l} (1-S_i^q)T_0 = S_i^q Q_i + (1-S_i^q)TB_i$$ \hspace{0.5cm} (13)

where $S_i^q$ is a switch with a value of either 1 for a specified heat input boundary condition or 0 for a specified base temperature boundary condition for the $i^{th}$ pin.

The above equations (13) and (14) comprise a set of $N \cdot (O + 1)$ equations for a similar number of unknowns ($q$ for $O$ segments on each of $N$ fins plus $T_0$ for $N$ fins). This set of equations can be written in matrix form and solved using standard matrix inversion methods to determine the heat flow and temperature distribution within the row of fins. These equations were implemented in a commercial mathematics code MathCad [16]. Correlation equations described earlier for the friction factor, Nusselt number and thermal wake for circular pin fins were implemented in the model. The solution results for $q$ and $T_0$ can be used with equations (10) and (11) to determine the fin temperature and adiabatic temperature for each segment of each fin.
becomes non-uniform so temperature predictions depart from the 1-D fin model.

Moving a step further we developed the model to represent a heat sink having a conducting base and a two dimensional array of fins. Heat transfer from each row of fins is treated as before but each fin is now connected to a heat sink base of finite thickness. Heat conduction within the base is solved using a standard finite volume approach [17] with boundary conditions supplied on the lower surface of the heat sink base. A simple version of base conduction was implemented by setting the size of each finite volume equal to the volume of the base under each fin, i.e., \( S_T \times S_L \times tb \). The base temperature \( T_0 \) for a fin is now the base temperature at the center of the finite volume below that fin. Thermal conductance terms connect the temperature in a control volume to adjacent control volumes in the \( x \) and \( y \) directions and to the boundary conditions specified on the lower surface of the heat sink base. The heat conduction equation for the base control volume below the \( j^{th} \) pin in the \( f^{th} \) row can be written as follows:

\[
2(C_y + 2C_x + 2S_{ij}C_j)T_{f,i,j,0} + \sum_k q_{i,j,k} \quad (14)
\]

\[
- C_x T_{i-1,j,0} - C_y T_{i,j-1,0} - C_y T_{i,j+1,0} + 1
\]

\[
= S_{i,j}q_{i,j} + (1 - S_{i,j})C_y T_{f,i,j}
\]

where the conductance terms are \( C_x = \frac{\lambda_m S_{L} b}{S_L} \), \( C_y = \frac{\lambda_m S_{T} b}{S_T} \), and \( C_z = \frac{2\lambda_m S_{L} S_{T}}{tb} \).

An additional resistance \( R_c = \left( \frac{L}{2\lambda_m D} \right) \) is added to the term \( \left( \frac{1}{\lambda_m D} \right) \) in equation (13) to account for conduction between the position at which \( T_0 \) is now computed and the base of the actual fin. This term includes the constriction resistance at the base of the fin. The heat transfer from the top surface of the heat sink base around the fins is included in the convective heat transfer expression for the first segment of the fin. The heat transfer coefficient on the base area was assumed to be 0.5 times the heat transfer coefficient on the surface of the fins based on data reported by Metzger et al. [18]. The constant of proportionality will mainly affect the thermal resistance predictions for heat sinks with sparse pin fin arrays where heat transfer from the wall area is significant. This wall heat transfer coefficient is very difficult to measure experimentally and its dependence on the array geometry, aspect ratio \( (L/D) \) of the pins, Reynolds number etc. is not known sufficiently and is a candidate for further study, maybe using CFD. The heat transfer from the fin tips has been left out of the present model but it can be easily included in the expression for the last segment of the fin if the fin tips are exposed to air flow.

**Extensions of the Heat Sink model**

Although we have focused on a simple transverse flow aligned with in-line rows of pin fins to develop the model, the approach presented here is readily extendable to non-orthogonal transverse flows including impinging flows. This would be done by partitioning the flow field into streamtubes (see [7]) which delineate the corresponding thermal wake linkages between segments, representing new effective pin rows, which may even be curved. Equation (12) provides the means to implement any “flow stream connectivity” implied by the flow streamtubes that describe the flow path through the array. If the wake function correlations are not available, a suitable first step is to set the wake function to unity.

Other types of fins such as interrupted plate, elliptical, wing shaped etc. can be modeled using the same heat sink model by simply providing the applicable correlation equations for Nusselt number and friction factor, e.g. those described by Muzychka and Yovanovich [19] for offset strip fin arrays.

**MODEL RESULTS AND DISCUSSION**

The heat sink model was compared against the experimental data of Jonsson and Moshfegh [13] for a heat sink consisting of a 9 x 9 array of 10 mm long 1.5 mm diameter pin fins arranged on a square grid with a pitch of 6.5 mm. We assumed a base thickness of 4 mm since this was not provided in [13]. Pressure drop and thermal resistance predictions were made for a fully ducted configuration for approach velocities ranging from 1.7 to 9 m/s corresponding to a Reynolds number range from about 200 to 1100.

CFD simulations were conducted using Icepak [14] for the above heat sink geometry using three levels of grid refinement. Even at the finest grid which used ~450,000 control volumes the CFD results were not grid independent. Further grid refinement was not conducted because the computational array sizes exceeded the physical memory on the desktop PC (~512 MB). Each CFD simulation took 44 minutes to complete.

**Figure 10. Model predictions for a pin fin heat sink compared to experimental data of [13]**

Figure 10 shows that the predictions from the model are in excellent agreement with the experimental data while the CFD simulations show considerable differences. For the model, the excellent agreement with the pressure drop data is to be expected since the data itself was used to determine the coefficients for the friction factor model (shown in Table 3). However, the good agreement over the whole velocity range shows the good correlation that is achieved between the Reynolds number and the friction factor through equation (4).


